

Metallic conduction, apparent metal-insulator transition and related phenomena in two-dimensional electron liquid

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Summary. — The paper introduces a reader to a relatively young field of the physics of strongly interacting and disordered 2D electron system, in particular, to the phenomena of the metallic conduction and the apparent metal-insulator transition in 2D. The paper briefly overviews the experimental data on the electron transport, magnetotransport, and spin-magnetization in 2D and on the electron-electron interaction effects.

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1. – Introduction

Resistivity of materials around us ranges by more than 30 orders of magnitude, from 10^{20} to 10^{-12} Ohm m. The variety of materials are classified as Metals (M) and Insulators (I), which are two distinct classes of matter only in the $T \rightarrow 0$ limit. The principle difference between the two classes is in the character of the electronic wavefunctions, which are spatially localized in insulators and extended in metals.

It is well-known, that subjected to even insignificant changes of pressure, dopant density, etc. many materials exhibit transformations between the two states [1]. The M-I transitions (MIT), unrelated to changes in the lattice structure and symmetry, are of a

special interest because they are considered to be continuous Quantum phase transitions, occurring at $T = 0$. The M-I transitions usually take place as density of electrons decreases; the latter necessarily leads to the increase of the effective strength of $e - e$ interaction. A dimensionless ratio r_s of the potential (Coulomb) energy to kinetic (Fermi) energy is often used to quantify interaction in 2D, $r_s \propto n^{-1/2}$ with n being the electron density [2]. On the other hand, the decrease in the density is accompanied with the increase in the effective disorder, particularly, due to the weakening of the screening of potential fluctuations. Thus, both, interactions and disorder are important in the vicinity of the MIT; the separation and interpretation of their effects represents a hard task [3].

In the limit of strong interactions $r_s \rightarrow \infty$ and zero disorder, the groundstate of the 2D system is believed to be the Wigner crystal (WC) of electrons. Correspondingly, at zero temperature and zero disorder, for 2D system, there is at least one critical point on the interaction axes, quantum melting of the Wigner crystal [4, 2]. A weak disorder is expected to stabilize the WC [5, 6]. As disorder increases further, WC loses long-range order and crosses over to a localized phase. Thus, in the limit of strong disorder and weak interactions, the ground state of the system is built of the single-particle localized states [7]. The experiments always take place at non-zero disorder and temperature. The subject of this paper is related to phenomena which occur due to the combined action of strong inter-electron interactions and disorder.

The outline of this paper is as follows: Sections 1 and 2 introduce a reader to the field and briefly review the main results in the semiclassical and quantum frameworks, correspondingly. Section 3 describes major results on transport in 2D systems, which are discussed in terms of the apparent metal-insulator transition in 2D. Section 4 describes quantitative studies of the effects of electron-electron interaction; the experimental results are compared with available theories in various regimes.

1.1. Mott semiclassical picture of the MIT.

At finite temperature, or for finite system size L , the metal and insulator states are not well-defined; one can be misled by attempting to distinguish these two states according to their conduction. In fact, the two states are inherently related with two distinct types of electron wavefunctions, localized and extended ones. The latter are classified according to the localization length ξ : if $\xi > (r_2 - r_1)$ the waves are extended (metal) and $\Psi \propto \cos(kr)$; if $\xi < (r_2 - r_1)$ the waves are localized (insulator) and $\Psi \propto \exp(-r/\xi) \cos(kr)$. Note, the electron spacing, $(r_2 - r_1) \sim \lambda_F = 2\pi/k_F = 1120 (10^{11} g_v / 2n)^{1/2} \text{ \AA}$ (the valley degeneracy $g_v = 2$ corresponds to (100)Si-crystal plane).

For the extended states, when $\lambda_F \ll l < L$ (with l being the mean free path) the electron motion results in a classical Drude conduction $\sigma = ne^2\tau_p/m = ne^2l/v_F m$. As disorder increases, l decreases. However, l can not be less than λ_F (the so called Ioffe-Regel criterium (1960)): $l_{\min} = \lambda_F = 2\pi/k_F$. Thus, we obtain the minimum metallic conductivity [1] $\sigma_{\min}(l_{\min}) = 2\pi e^2/h$ in 2D. We used here $k_F^{2D} = \sqrt{2\pi n}$. Note, that $h/e^2 = 25812.7 \text{ Ohm}$. In this semiclassical picture [1], the M-I transition is caused by the disappearance of the localized (or, vice versa, extended) states.

1.2. Basics of the 2D semiconducting devices

The 2D electron and hole systems discussed in this lecture consist of the bulk carriers confined in a two-dimensional potential well; the latter is formed at the interface between two semiconductors or semiconductor/insulator interface. Figure 2 schematically shows crossview of the silicon metal-oxide-semiconductor (MOS) structure [2]. The 2D layer of electrons is confined at the interface between Si and SiO₂, grown at the top of the Si-substrate. On the top of the insulating layer, a thin metallic film (gate) is deposited. Throughout this lecture, the electrons and holes are the same quasiparticles and for simplicity will be often called “electrons”.

When the positive voltage V_g is applied to the gate (relative to the source, or drain contact, or relative to the Si-bulk), the conduction E_C and valence E_V band edges bend down as Figure 2 shows. For a sufficiently high gate voltage, the bottom of the conduction band decreases below the bulk Fermi level and the resulting triangular potential well starts populating with electrons [2]. At room temperatures, when the bulk Si is conducting, redistribution of the electrons between the bulk and the 2D layer occurs in accord with the Poisson equation [2]. When the gate voltage is applied or changed at low temperatures, the bulk conduction is frozen-out, the equilibrium with bulk is not achieved, and the required electrons come into (or out) the 2D layer from the potential contacts. The contacts are lithographically defined dopant diffusion areas with high concentration of electrons supplied by the dopants.

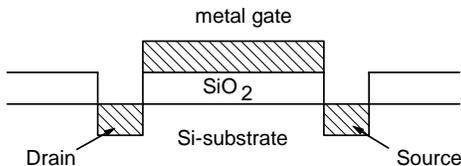


Fig. 1. – Schematic structure of a silicon field effect transistor. The 2D electron gas is confined at the Si side of the Si/SiO₂ interface. Drain and source are diffusion areas used as ohmic contacts to the 2D layer. Metallic gate is deposited on the top of the SiO₂ layer [2].

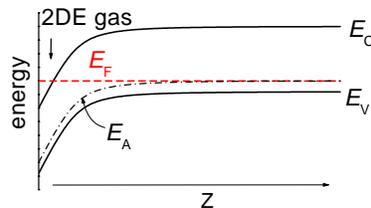


Fig. 2. – Schematic energy band diagram of the Si-MOS structure. E_C (E_V) is the bottom (top) of the conduction (valence) band, E_A -bulk acceptors energy level, E_F - Fermi level, Z -axes is directed to the bulk, perpendicular to the surface.

For our purpose, it is essential only that the layer of electrons confined at the interface is two-dimensional, and that its charge Q is controlled by voltage V_g applied to the gate: $Q \equiv ne = C(V_g - V_t)$, with V_t , the so called threshold voltage. Thus, the Si-MOS system may be viewed as a plane capacitor, whose one plate is formed by the 2D layer of electrons whereas the gate serves as another plate.

2. – Quantum transport at zero field

2.1. Various types of transport: Delocalized states: diffusive and ballistic transport. There are two regimes of the metallic-like transport: diffusive for $l < l_\varphi < L$, and ballistic regime for $l > L$. The phase breaking length $l_\varphi = \sqrt{D\tau_\varphi}$ (with τ_φ , the phase breaking

time, D - diffusion coefficient) is related with large changes of electron energy and is similar (not equivalent though) to inelastic length. The relevant scattering processes for τ_φ at high temperatures are (i) electron-phonon scattering, and for low temperatures - (ii) electron-impurity and (iii) electron-electron scattering. In general, $\tau_\varphi \propto T^{-p}$ with $p = 1 - 2$, depending on the dominating scattering process. Since electron-electron collisions conserve the total momentum, the transport time l_p (momentum relaxation) is determined by the two former processes and is insensitive to electron-electron scattering.

2.2. Localized states.

If an energy barrier Δ separates the energy of the electron states from conduction (or valence) band, and the wavefunctions of localized states don't overlap, the conduction occurs via temperature activated transitions whose probability is $\sigma \propto \exp(-\Delta/T)$. When the probability of the temperature activation is too low (e.g., for high Δ , or for low temperatures), transport occurs via elastic tunneling between the localized states. This is the so-called hopping conduction regime, where the characteristic temperature dependence of the conductivity is $\sigma \propto \exp((-T_0/T)^p)$, where $p = 1/2$ or $1/3$, depending on a specific model [7]. Usually, hopping conduction dominates at low temperatures.

2.3. Electron's Phase Coherence and Transport.

In the quantum-mechanical picture, the electron waves propagate and interfere. The interference gives rise to quantum corrections to the conductivity [8, 9]. In 2D system of non-interacting electrons (Fermi-gas), as $T \rightarrow 0$:

$$(1) \quad \sigma = \sigma_D - \frac{e^2}{\pi h} \ln(\tau_\varphi/\tau) \sim \sigma_D + \frac{e^2}{\pi h} \ln(T\tau),$$

where σ_D is the semiclassical Drude conductivity. The single-particle interference thus may be viewed as a quantum "backscattering". Weak logarithmic dependence is a typical attribute of the 2D system at high electron density (low r_s), and may be observed at high conductivity $\sigma \gg e^2/h$ in the low-temperature diffusive regime $T\tau \ll 1, \tau \ll \tau_\varphi$ (see fig. 3).

2.4. Suppression of the weak localization in H_\perp fields

The quantum interference corrections originate from small loops of self-intersections on the trajectories, which quantum particles propagate in both directions, due to the time reversal symmetry [8, 9]. The amplitudes of the wavefunction for a particle passing the loop clockwise and counterclockwise acquires additional factors: $A_1 \rightarrow A_1 \exp(\pm i\pi\Phi/\Phi_0)$ with $\Phi_0 = h/e$ being the flux quantum and Φ the magnetic flux through the loop. The magnetic field applied perpendicular to the 2D plane of motion, destroys the interference by reducing the probability for a particle to return. The interference breaks down when the phase difference $\Delta\varphi$ becomes ~ 1 :

$$(2) \quad \Delta\varphi \sim \frac{HD\tau_\phi}{\Phi_0} \sim 1 \quad H \sim \frac{h}{eD\tau_\varphi}$$

The "negative magnetoresistance" effect is a tool to determine τ_φ [9, 11].

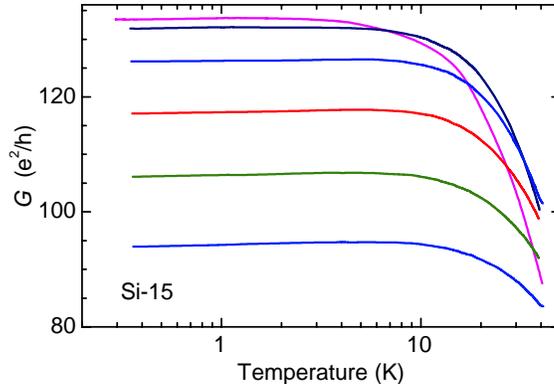


Fig. 3. – $\sigma(T)$ dependences for high densities [10] (from top to bottom, in units of 10^{11} cm^{-2}): 32.1, 42.94, 48.4, 53.8, 64.7, 75.6, 86.5.

2.5. Single-particle scaling theory of localization (E. Abrahams et al., 1979) [12]

The one-parameter scaling theory considers the dependence of the conductivity on the system size. When the size L is changed, the effective disorder is changed; it is assumed that the only measure of this is the conductance G . The above assumption is equivalent to introducing a function β such that

$$(3) \quad \beta = \frac{L}{G} \frac{dG}{dL} = \frac{d \ln G}{d \ln L},$$

where β is a universal function of G solely.

One can consider how the scaling hypothesis fits various obvious limits:

- (1) In the Ohm's law region ($G \gg e^2/h$), $\beta = d - 2$ and the conductance does not depend on L for the two-dimensional case, $d = 2$.
- (2) If the states are localized, $G(L) \propto \exp(-L/\xi)$. Therefore, $\beta(G) = \ln(G/G_0)$.

The two distinct limits enable to plot the asymptotes of β in fig. 4 a. One may expect that between these two limits, β should vary smoothly, as shown schematically in fig. 4 a.

Note: It is not necessary to vary L in order to move along the scaling trajectories. For finite temperatures, l_φ causes inelastic cut-off, so that the temperature tunes the effective sample length.

In $d = 3$, as fig. 4 a shows, the scaling function $\beta < 0$ for very small G (strong disorder). As conductance increases, β passes through zero at a certain critical value G_c . This is a repulsive point or an “unstable critical point” in the renorm-group terminology. For low initial $G < G_c$, increasing the size of the system will cause G to decrease and the behavior of the system approaches that for the localized states. In contrast, when the initial conductance is high, $G > G_c$, increasing the size of the system will lead to the Ohmic regime (metallic conduction). Such behavior and the corresponding metal-insulator transition are consistent with the Mott' semiclassical picture described above.

In lower dimensions, $d < 3$, β is always negative and all states are localized. As system size L increases, the system moves to the strongly localized regime. The scaling

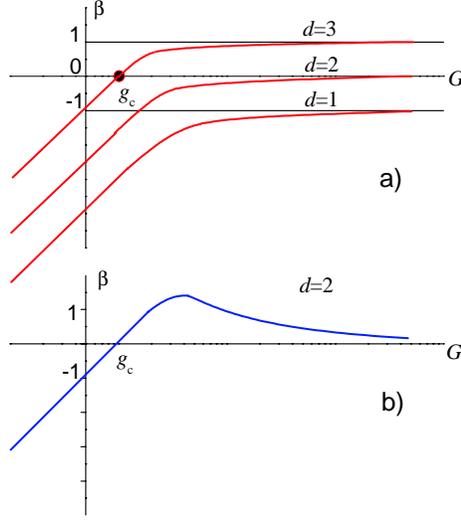


Fig. 4. – a) Schematic behavior of the scaling function for non-interacting electrons in different dimensions d . Bold dot depicts the critical point of the M-I transition for $d = 3$. b) Spin-orbit scattering case for $d = 2$.

function describes a crossover from weak localization (localization length $\xi > L$) to strong localization ($\xi < L$) regime.

It follows from the one-parameter scaling theory that “there is no true metallic conduction in $d < 3$ ” [12]. For $d = 1$, this conclusion is obvious since the negative scaling function has substantially large amplitude $|\beta| \geq 1$. In accordance with eq. (4), this causes the conductance to decrease quickly as L increases, approaching that for the strongly localized case. However, the $d = 2$ case is more tricky: for high $G \gg 1$, the scaling function is so close to zero, that minor correction may potentially change the sign of β , therefore the situation requires a special attention.

a) Spin-orbit scattering case (Hikami, Larkin, Nagaoka (1980)). The electron interference is destroyed by spin-orbit random scattering and the weak localization “back-scattering” correction vanishes [13, 8, 9]. More over, the spin-orbit scattering induces a positive correction (“forward scattering”) so that the β -function becomes $\propto +1/2G$ for high G , as schematically shown in fig. 4 b. Note, that this results is valid only for a hypothetical 2D system of non-interacting particles.

b) Interaction quantum corrections in the diffusive regime. Altshuler-Aronov corrections take into account $e - e$ interactions in the $T\tau \ll 1$ limit [8, 14]:

$$(4) \quad \delta\sigma = -\frac{e^2}{2\pi^2\hbar} \left[1 + 3 \left(1 - \frac{\ln(1 + F_0^a)}{F_0^a} \right) \right] \ln(\hbar/k_B T\tau),$$

where F_0^a is the Fermi-liquid interaction parameter, dependent on r_s (see further for more details). In the $F_0^a \rightarrow 0$ limit, the total quantum correction $\delta\sigma_{loc} + \delta\sigma_{so} + \delta\sigma_{ee}$ is usually negative (backscattering). However, when $F_0^a < 0$ and $|F_0^a|$ is large, the total

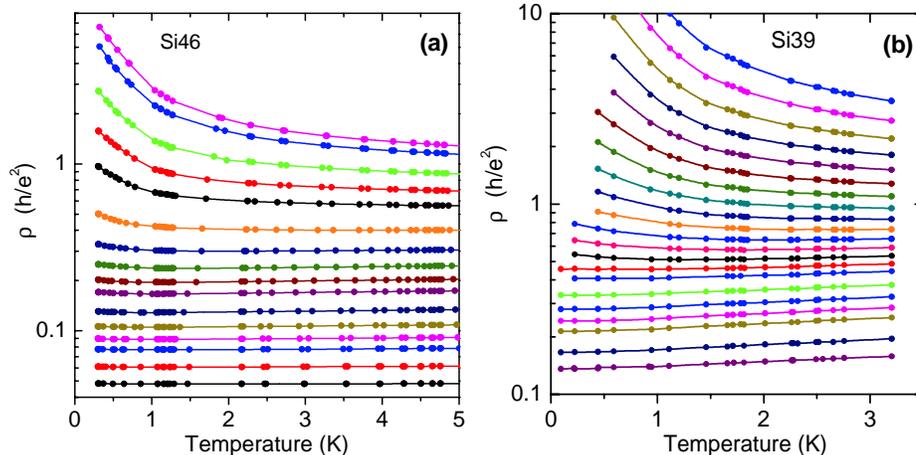


Fig. 5. – Resistivity vs temperature for disordered Si-MOS samples: (a) with peak mobility $\mu = 1500 \text{ cm}^2/\text{Vs}$, (b) $\mu = 5100 \text{ cm}^2/\text{Vs}$. The densities (in units of 10^{11} cm^{-2}) on the left panel span from 3.85 to 37.0 (from top to bottom), on the right panel – from 1.95 to 5.44.

quantum correction may lead to delocalization (“forward scattering”). This case will be discussed in Sec. 4 in more details.

c) Interaction quantum corrections to the transport in the ballistic regime $T\tau \gg 1$ have been calculated in Refs. [15, 16]; they are discussed in Sec. 4.

3. – An apparent MIT in 2D

A great body of experimental data accumulated since 1979 support the conclusions of the scaling theory [17, 18]. An example is shown in fig. 5 a. The uppermost curves in figs. 5 show that the resistivity for a very disordered (low-mobility) Si-MOS sample at low densities exhibits an exponentially strong increase with cooling. This is consistent with motion to the left hand side along the logarithmic part of the β -function in fig. 4 a. For higher densities (lower curves in fig. 5), the exponential increase of the resistivity is replaced by a weak temperature dependence. The latter one at very high carrier densities in Si-MOS or in GaAs samples is logarithmic (see, e.g. fig. 3) and consistent with the theory ($d\rho/dT > 0$).

As sample mobility increases, and, correspondingly, the density of the crossover (from the exponential to logarithmic T -dependence) decreases [19], the temperature derivative $d\rho/dT$ obtains “wrong” positive sign (compare figs. 5 a, b, and fig. 6). This effect was noticed more than two decades ago [20, 21, 22] and discussed in terms of the temperature-dependent screening [23, 24, 25]. As will be shown below, the sign of $d\rho/dT$ is determined by the interaction parameter F_0^a , which in its turn, depends primarily on the electron density.

The strong indication that the prediction of insulating states in 2D may not be universally valid was obtained in 90s in studies of the quantum Hall liquid to insulator

transitions [26, 27, 28]. Extended states, which in high magnetic field B are located at centers of Landau bands, were found experimentally to merge and remain in a finite energy range as $B \rightarrow 0$. This behavior is not expected within the scaling theory, which predicts that the extended states “float up” in energy in that limit [29, 30, 31]. A serious challenge to the scenario of the insulating ground state in 2D arose in 1994 and the years following, when metallic behavior of electrical transport in zero magnetic field was observed in high mobility Si MOSFET structures [32, 33] (see fig. 6).

Subsequently, qualitatively similar metallic states were found in other high mobility 2D systems (for an extensive bibliography, see Ref. [34]). These include p - and n -Si/SiGe- [35, 36, 37], p -GaAs- [38, 39, 40, 41, 42], n -GaAs- [43, 44], n -AlAs-heterostructures [45], inverted Si-on-insulator structures [46] and back-gate biased Si-MOS structures [47]. The most important findings of these studies can be summarized as follows:

Features in zero magnetic field:

- If the density of 2D carriers n is larger than a sample-dependent value n_c , the $\rho(T)$ dependence is metallic-like, i.e. $d\rho/dT > 0$ (see fig. 6). The changes in ρ can be almost an order of magnitude (in high mobility Si-MOSFETs [32]).
- The strong metallic-like $\rho(T)$ -dependence is characteristic of a wide range of densities rather than of the critical regime; it sets in for relatively high temperatures $T < T_F$ in the ballistic rather than diffusive regime [48] (see fig. 7).

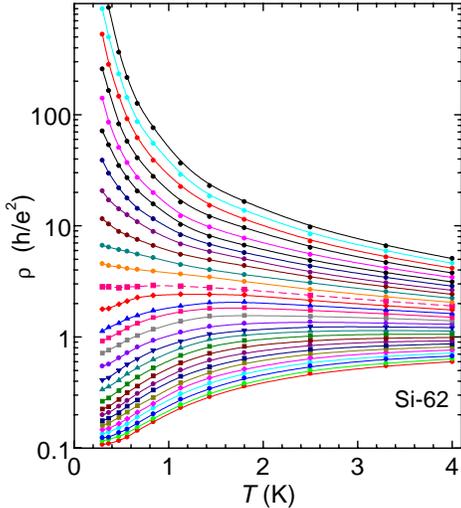


Fig. 6. – Resistivity vs temperature for HIGH mobility Si-MOS sample [51] in the critical regime $n \approx n_c$. Electron density decreases from 1.326 in steps of 0.0218 (from bottom to top) and in unites of 10^{11}cm^{-2} .

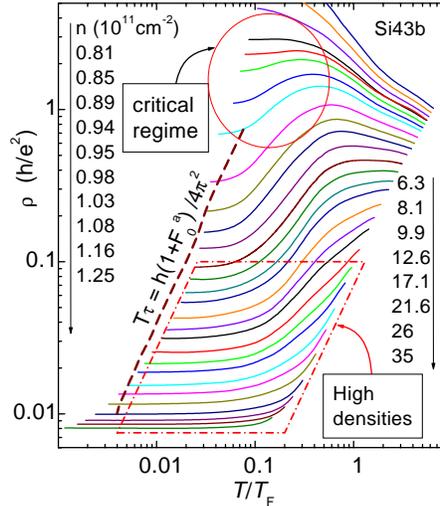


Fig. 7. – Resistivity vs T/E_F in a wide range of densities. The diffusive/ballistic border for $T\tau$ [15] is calculated based on the data [48] with renormalized m^* .

- When $n < n_c$, both the sign ($d\rho/dT < 0$) and the functional (exponential) form of the $\rho(T)$ dependence are characteristic of the insulating behavior (see figs. 5,6).
- At much higher densities ($n > 30 \times n_c$), the derivative $d\rho(T)/dT$ is metallic (positive) at higher temperatures $T \sim T_F$, but ρ exhibits a slow (logarithmic) “insulating” upturn at lower T , consistent with eq. (2) [10, 49, 50] - see fig. 3. Qualitatively, conduction at high density is consistent with the weak localization picture.
- The $\log(R(T))$ data show a reflection symmetry with respect to n_c [32, 33] (see fig. 6), with exclusion of the low-temperature data, which in the $T \rightarrow 0$ limit diverge for $n < n_c$, but do not fall to zero for $n > n_c$. This signifies that at intermediate temperatures, the $\rho(T)$ -dependence is empirically represented by a logarithmic function $\rho(T) \propto \exp[\pm(T_0/T)^p]$ with $p \sim 1$ and the sign equal to that of $(n_c - n)$ [51]; such dependence obviously fits the one-parameter scaling function in the vicinity of a critical point shown in fig. 4 b.
- For high-mobility samples, the resistances of the metallic and insulating phases, measured as a functions of T and n , exhibit scaling properties with respect to T/T_0 [32, 33, 51] demonstrated in fig. 8. The energy scale T_0 goes to zero on both sides of the transition; such behaviors are reminiscent of a critical point of a continuous quantum phase transition [52]. This interpretation, however, seems to work only for intermediate temperatures and fails in the $T \rightarrow 0$ limit. For a more detailed review and phenomenological discussion of the experimental data in the critical regime, see ref. [53].

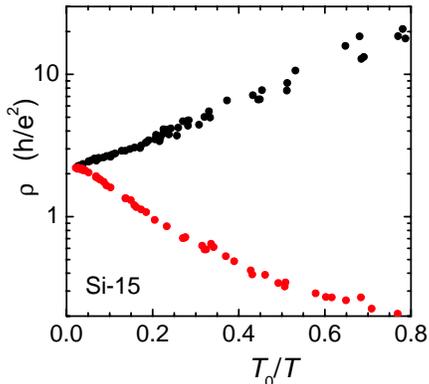


Fig. 8. – Resistivity for high- μ sample scaled vs T_0/T [51]

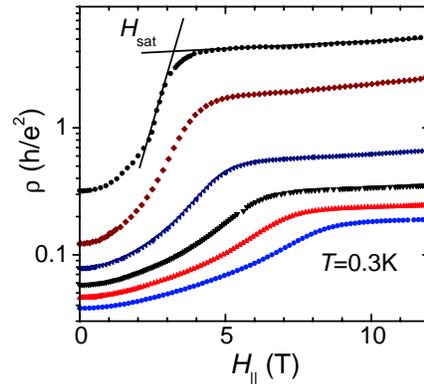


Fig. 9. – $R(H_{||})$ dependences for different densities [59].

Features in the in-plane field

- In Si-MOSFET and p-GaAs structures, ρ increases strongly with $H_{||}$ [54, 55, 56, 57, 58, 59, 60] and shows saturation at a field, which is approximately equal to that for complete spin polarization [61]; this is most sharply pronounced in Si-MOS structures [56, 55, 54, 59].

- As B_{\parallel} couples to spins rather than to orbital motion, it follows that the spin-related mechanism of interactions (i.e. exchange) is of major importance. More specifically, when H_{\parallel} field switches off the spin-degrees of freedom, the $R(T)$ dependence flattens and MIT disappears [62, 63].

As n_c is quite small ($\sim 10^{10} - 10^{11} \text{ cm}^{-2}$), the corresponding r_s values are large, an order of 10; therefore, it is reasonable to assume the $e - e$ interaction to be one of the major driving forces in the above listed phenomena. In the following section, the $e - e$ interaction effects will be discussed in more detail. We shall show, in particular, how the listed features find a natural explanation within the frameworks of the quantum interaction corrections at intermediate temperatures.

4. – Quantitative studies of the electron-electron interactions

4.1. Fermi-liquid renormalization of electron parameters in 2D systems

The Landau Fermi-liquid (FL) theory [64, 65] introduces a number of interaction parameters to describe the interacting system in terms of the renormalized g^* -factor Landé, effective mass m^* etc.:

$$(5) \quad F_0^a = \frac{2}{g^*} - 1, \quad F_1^s = 2 \left(\frac{m^*}{m_b} - 1 \right).$$

Recently, these parameters have been determined experimentally in a wide range of densities, in measurements of the quantum oscillations of conductivity (Shubnikov-de Haas (SdH) effect) in tilted or crossed weak magnetic fields [57, 66, 67, 68]. In these experiments, the renormalized spin susceptibility χ^* is obtained with no assumptions:

$$(6) \quad \chi^* = g^* g_b m^* \mu_B \frac{e}{h}, \quad \frac{\chi^*}{\chi_b} = \frac{g^* m^*}{2m_b} = \frac{(F_1^s + 2)}{(F_0^a + 1)},$$

where g_b , m_b , and χ_b are the corresponding band values. Figure 10 shows the r_s -dependence of the measured renormalized χ^* and m^* -values [57, 66]; the renormalized g^* -values are obtained from these two sets of data as χ^*/m^* . Other experimental approaches consist in scaling the $\sigma(H/H_0)$ -data for different densities [69, 70, 71], or in fitting the $\sigma(T)$ and $\sigma(H_{\parallel})$ dependences to the calculated quantum corrections [15], using F_0^a (and in some cases, also m^*) as fitting parameter [72, 73, 74, 75].

The resulting F_0^a values obtained for 2D electrons in GaAs [68] and in Si-MOS samples [71, 70, 66, 73, 75, 76, 72] qualitatively agree with each other [77] but differ substantially from the data for 2D holes in GaAs [74], and, especially, from the data taken for low carrier density [42]; the reason for this discrepancy is not clear yet [77].

Figure 10 shows a strong increase in χ^* , m^* , and g^* with r_s , which should significantly affect transport at low densities. The growth in χ^* might reflect a tendency to either ferromagnetic or antiferromagnetic transition; besides, within the FL-description eq. (5), the g -factor should diverge as $F_0^a \rightarrow -1$. Therefore, a separate, interesting question is

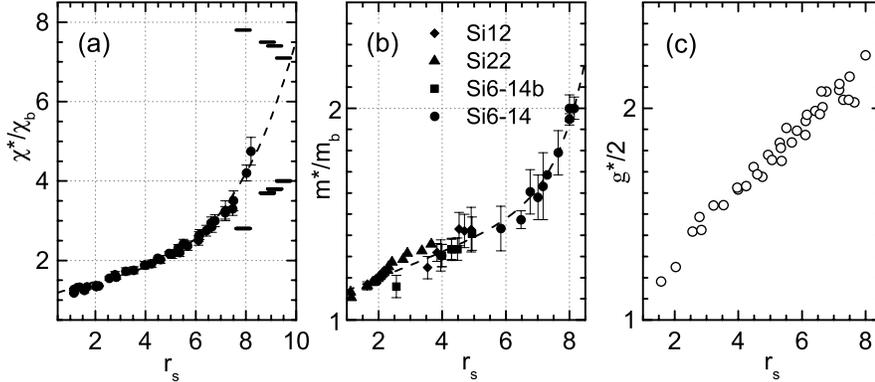


Fig. 10. – Renormalized spin susceptibility (a), effective mass (b), and g^* -factor (c) measured with Si-MOS samples [66, 77].

whether or not χ^* , m^* and g^* diverge as r_s increases, and whether the divergency (if any) occurs at the MIT. Numerical calculations can not give an unambiguous answer [78, 79, 80] for the single-valley system, whereas the 2D valley system is expected to be more stable and to remain unpolarized up to the WC phase.

As a result of the renormalization, the energy scale E_F diminishes. However, in the vicinity of the transition (i.e. at the sample-dependent density $n = n_c$), m^* does not diverge [81, 77]; this may be easily seen, e.g., from non-vanishing amplitude of the SdH oscillations measured at $T = 0.03$ K for r_s values up to 9.5 [81]. Thus, the MI-transition is not caused by simply vanishing the E_F value and the quasiparticles. In Refs. [71, 69] the in-plane magnetoresistance $\rho(H_{\parallel})$ data have been discussed in terms of a developing ferromagnetic instability at or very close to n_c . However, direct measurements of the (a) magnetization of 2D electrons [82], (b) period, and (c) sign of the SdH oscillations [81, 77] do not confirm such possibility: the spin susceptibility remains finite at $n \geq 7.7 \times 10^{10} \text{cm}^{-2}$, the density which is lower than the critical n_c -value for many samples.

The strong enhancement of the weak field spin susceptibility was predicted also by the renorm-group (RG) theory [14, 83, 84]; however the RG-theory is based on consideration of the diffusive modes of electron-electron interaction in the regime $T\tau \ll 1$, whereas the renormalized parameters (Fig. 10) are measured in the ballistic interaction regime $T\tau > 1$ (for a more detailed discussion, see Section 4.3).

4.2. Implementation of the measured FL parameters to the metallic-like transport.

A considerable progress has been also achieved in the theory of quantum transport: in Refs. [15, 16] the interaction corrections to the conductivity have been calculated beyond the diffusive regime, in terms of the FL interaction parameters. In this section, we make a comparison of the data with the theory and conclude that the “metallic” drop of conductivity with cooling in the regime $\sigma \gg e^2/h$ can be accounted for by the

interaction effects in electron “liquid” at temperatures which correspond to the ballistic regime $T\tau > 1$. This observation suggests that the anomalous “metallic” conduction in 2D, at least for densities not too close to the critical density, is the finite-temperature phenomenon rather than the signature of a new quantum ground state.

The theory [15] considers backscattering of electrons at the scattering centers and at the Friedel oscillations of the density of surrounding electrons. The interference between these scattering processes gives rise to the quantum corrections, which are calculated to higher orders in the interactions and leading order to the temperature. The interference gives rise to quantum corrections to the Drude conductivity (in units of $e^2/\pi\hbar$):

$$(7) \quad \sigma(T) - \sigma_D = \delta\sigma_C(T) + 15\delta\sigma_T(T).$$

$$(8) \quad \begin{aligned} \text{Here} \quad \delta\sigma_C &= x \left[1 - \frac{3}{8}f(x) \right] - \frac{1}{2\pi} \ln \left(\frac{E_F}{T} \right) \quad \text{and} \\ \delta\sigma_T &= A(F_0^a)x \left[1 - \frac{3}{8}t(x, F_0^a) \right] - B(F_0^a) \frac{1}{2\pi} \ln \left(\frac{E_F}{T} \right) \end{aligned}$$

are the interaction contributions in the „charge” (exchange term and singlet part of the correlation terms) and triplet channels, respectively; $x = T\tau k_B/\hbar$, $A(F_0^a) = F_0^a/(1 + F_0^a)$, and $B(F_0^a) = 1 - \ln(1 + F_0^a)/F_0^a$. The prefactor 15 to $\delta\sigma_T$ reflects enhanced number of triplet components: two valleys and two spins produce 4 pseudospin components; interaction of two particles involves $4 \times 4 = 16$ channels, one of them being singlet.

Diffusive regime, $T\tau \ll 1$. For large $|F_0^a|$ and $F_0^a < 0$, $\delta\sigma(T)$ becomes positive. E.g., for $F_0^a = -0.3$ (i.e. $g^* \approx 3$)

$$(9) \quad \delta\sigma = \frac{1}{2\pi} (1 - 15 \times 0.2 + 1) \ln T = -\frac{1}{2\pi} \ln T.$$

Thus, due to valley degeneracy, the quantum correction is constructive and produces a “forward scattering”.

Ballistic regime, $T\tau > 1$. The quantum correction $\delta\sigma(T)$ is quasilinear in T :

$$(10) \quad \delta\sigma(T) \approx \left[1 + 15 \frac{F_0^a}{1 + F_0^a} \right] (T\tau)$$

Comparison of the theory with experiment at high $G \gg e^2/h$ (see Fig. 7).

The terms in eq. (7) are functions of $T\tau/\hbar$ and F_0^a . The momentum relaxation time τ may be found from the Drude resistivity $\rho_D \equiv \sigma_D^{-1}$ using the renormalized effective mass m^* determined in Ref. [66]. Thus, the comparison shown in fig. 11 has no fitting parameters, beyond σ_D . The determination of the latter value is transparent [73], it is obtained by a quasilinear extrapolation of the experimental $\sigma(T)$ data to $T = 0$, by setting the logarithmic (diffusive) terms to zero. Almost entire temperature range in fig. 11 belongs to the ballistic regime. In view of the quantitative agreement of the

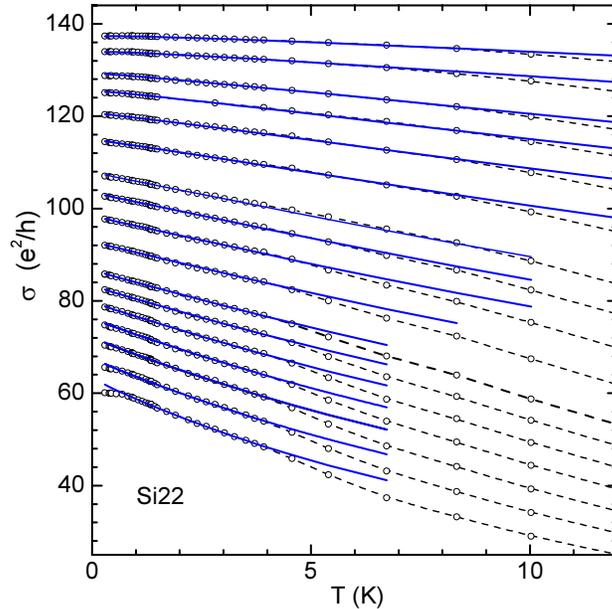


Fig. 11. – Conductivity for sample Si22 vs temperature [85]. The electron densities, from top to bottom are: $n = 5.7, 6.3, 6.9, 7.5, 8.1, 8.7, 9.3, 10.5, 11.7, 12.9, 14.1, 16.5, 18.9, 21.3, 23.7, 28.5, 35.7$ (in units of 10^{11}cm^{-2}). Dots represent the data, solid lines - the theoretical curves with F_0^a and m^* from Ref. [66].

experimental data with theory, we conclude that the initial slope of the $\sigma(T)$ dependence is well described by the quantum interaction corrections eqs. (7) with $F_0^a(r_s)$ and $m^*(r_s)$ values determined independently (see fig. 10). Several experimental teams came to a similar conclusion for p -GaAs/AlGaAs [75, 42] and Si MOSFETs [72, 76, 86, 73].

We conclude this section by listing the achieved results:

- Not too close to the critical regime, $\sigma \gg e^2/h$, the “metallic” strong T -dependence results from the quantum interactions at intermediate range of T ’s.
- As T decreases further, in the diffusive regime, $\sigma(T)$ is expected to vary $\propto \ln T$ with $d\sigma/dT < 0$ for sufficiently large g^* (large $|F_0^a|$) and large number of valleys. Than, for $T < T_0$ (determined by the intervalley h/τ_{vv} and spin-spin scattering rate h/τ_{ss}), the enhancement factor (15) should vanish and the localizing terms are expected to overwhelm.
- If the true metallic phase in 2D exists, it must of a non-FL type [52, 87, 88]. Several non-FL theoretical models have been recently suggested, such as phase separation model (Wigner solid inclusions in the FL) [89], an electron-paired state with broken U(1) symmetry [90], magnetic phase transition etc. However, so far no clear signatures of the non-FL behavior have been detected experimentally.

4.3. Transport in the critical regime, $\sigma \sim e^2/h$ and $n \approx n_c$

In view of the encouraging comparison with the theory of interaction corrections in the high density regime, it seems attractive to extend the comparison to the critical regime, where $\sigma \sim e^2/h$ (see fig. 7). Quantum corrections theory is inapplicable for large conductivity changes $\delta\sigma/\sigma \sim 1$ and in the vicinity of the critical point, where $\sigma \sim e^2/h$. The method now commonly in use is a generalization of the nonlinear σ -model theory, which has been developed by Finkel'stein [14] and by Castellani, Di Castro et al. [83, 91]. The renormalization-group method is an extension of the same diagrams of the interaction theory to the case of the strong disorder and interaction [92]. The RG equations describe renormalization of both, disorder (i.e. ρ) and interaction parameters as the length scale varies with T [14, 91, 93]:

$$(11) \quad \begin{aligned} \frac{d\rho}{dy} &= \frac{\rho^2}{\pi} \left[1 + 1 - 3 \left(\frac{1 + \gamma_2}{\gamma_2} \ln(1 + \gamma_2) - 1 \right) \right] \\ \frac{d\gamma_2}{dy} &= \frac{\rho(1 + \gamma_2)^2}{2\pi}, \end{aligned}$$

where $y = -\ln(T\tau)$, γ_2 is the Fermi-liquid amplitude for large angle scattering ($\gamma_2 \rightarrow -F_0^a/2$ in the weak-coupling limit), and ρ is in units of h/e^2 . The 1st term in the square brackets is the weak localization contribution, the 2nd term is the $e - e$ interaction correction in the singlet channel [8, 9], and the last term is the contribution from 3 triplet modes. Due to the difference in symmetry of the singlet and triplet wavefunctions, the singlet and triplet terms have different sign, and favors localization and delocalization, correspondingly. For two-valley system, factor 3 should be again replaced by 15, and the weak-localization term also becomes twice larger [94].

The above equations have a universal solution, which has been compared with experimental $\rho(T)$ data in Ref. [94]. We present in fig. 12 a similar comparison, which confirms a certain similarity between the $\rho(T)$ -data and the theory. There is a limitation though, the eqs. (11) are perturbative, i.e. are derived in the lowest order in ρ , therefore, the comparison with experimental data for $\rho(e^2/h) \sim 1$ can be only qualitative.

The application of the RG perturbative equations to the experimental data is based on the following assumptions:

- (i) the critical regime $n \approx n_c$ belongs to the diffusive domain of temperatures $T\tau \ll 1$;
- (ii) the $\rho(T)$ maximum signifies a turning point from localization to delocalization behavior due to the renormalization of the interaction parameter (primarily, γ_2) with T ;
- (iii) the decrease in ρ with cooling takes place mainly for the account of the strong growth in the interaction parameter γ_2 (and hence, F_0^a).

The first assumption does not agree well with the empirical diagram in fig. 7; therefore, its application would require to re-interpret the definition of the diffusive/ballistic border, i.e. to find a different interpretation for the $\tau(n)$ or $F_0^a(n)$ data in the critical regime. In Ref. [94], the assumption (ii) was presumed to be fulfilled in the most clean samples; however, some high mobility Si-MOS samples do not exhibit a pronounced $\rho(T)$ maximum

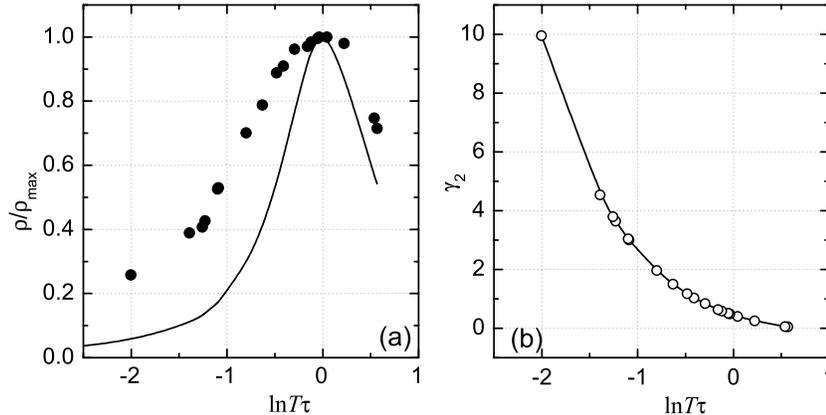


Fig. 12. – a) Comparison of the $\rho(T)$ data normalized to its maximum value ($1.35h/e^2$) with solution of the RG equations (solid line) eq. (11); b) Renormalized interaction parameter γ_2 [94], calculated from the 2nd Eq.(11) with $\rho(T)$ data shown on the left panel. The „0”-point at the $\ln(T\tau)$ scale corresponds to $T = 1.8\text{K}$. Sample Si15, electron density $n = 0.88 \times 10^{11}\text{cm}^{-2}$.

in the critical regime [95, 51] (see, e.g., fig. 6). Furthermore, the $\rho(T)$ data in the critical regime, as a rule, show a strong sample-dependent and non-universal behavior [51, 96]. It is possible though that the relevant disorder is related with short-range fluctuating potential at the interface which may favor inter-valley scattering and, thus reduce a number of the triplet terms in eq. (11) but is not significant for the mobility at low T .

Regarding assumption (iii), figure 12 b shows that the interaction parameter γ_2 , determined from the experimental data, starts growing at rather high temperatures, $\sim 1.8\text{K}$ for $n \approx n_c$; the latter behavior of γ_2 is necessary in the RG-theory to turn down the resistivity flow. Thus, the renormalization of γ_2 should occur in a quite accessible range of temperatures (0.3 - 1.8)K. However, the anticipated strong T -dependence of the renormalized spin susceptibility (eq. (6)) $\chi^*/\chi_b \propto g^*m^*/2m_b$ [83] is not observed experimentally at these temperatures in SdH-effect measurements: χ^* changes only within 2% in the above temperature range [77].

An alternative theoretical approach [52] suggests that the critical behavior in the vicinity of n_c is dominated by the physics of the insulating state, namely by the melting of the disordered Wigner crystal [88], electron glass [97], or an inhomogeneous state consisting of inclusions of WC into the 2D liquid [89]. Corresponding theories has not reached yet a fully predictive stage. We note that signatures of the glassy behavior were reported in Ref. [98].

4.4. Transport in the presence of the in-plane field

Figure 9 shows a typical behavior of $\rho(B_{\parallel})$. At low densities $n \sim n_c$, as field increases,

transport becomes temperature activated (hopping regime) [63]. When n approaches n_c , by definition, $n_c(H_{\parallel})$ tends to $n_c(H_{\parallel} = 0) \equiv n_c$, therefore, the range of magnetic fields where the magnetoresistance (MR) may be studied in the regime of “metallic” non-activated conduction, shrinks to zero.

Regime of high densities $n \gg n_c$, $G \gg e^2/h$. According to the theory of interaction corrections [15],

$$(12) \quad \delta\sigma(H_{\parallel}) \equiv \sigma(0, T) - \sigma(E_Z, T) \approx \frac{e^2}{\pi\hbar} f(F_0^a) \frac{T\tau}{\hbar} \left[K_b \left(\frac{E_z}{2T}, F_0^a \right) \right]$$

where $K_b \approx (E_z/2T)^2 f(F_0^a)$ in the low field limit $E_z/2T \ll 1$, and $K_b \approx (E_z/T) f_2(F_0^a)$ in the limit $E_z/2T \gg 1$ limit. Correspondingly, as field increases, $\delta\sigma(H)$ should increase initially $\propto H^2/T$, and then $\propto H$.

Experimental data, in general, show similar behavior [86, 66, 73] with a transition from parabolic to linear dependence; however, in contrast to the temperature dependence of ρ , the agreement with theory [15] is only qualitative [73]. In high fields, $g^*\mu H_{\parallel} \gg T$, the magnetoresistance deviates substantially from the theory, and the deviations are sample dependent [73]. As density decreases and field increases, the deviations of the measured MR from the theory grow and reach a factor of two at $n = 1.5n_c$ [73, 63]. The measured $\sigma(H, T)$ scales with temperature somewhat different from eq. (12), which predicts $\delta\sigma \propto -(H^2/T)$ in low fields.

Critical regime of densities $n \approx n_c$, and $\sigma \approx e^2/h$. As mentioned above, the magnetoresistance studies in the critical regime are restricted to the fields vanishing to zero as $n \rightarrow n_c$. Nevertheless, down to densities $n \approx 1.1n_c$. the magnetotransport can be safely studied in the regime $g^*\mu H_{\parallel} \ll T$. The experimental data in low fields scales as $\delta\sigma \propto -(H^2/T^p)$, where p increases from 1.1 to 1.6 as density decreases from $5n_c$ to $1.2n_c$. Comparing this empirical scaling law with that predicted by the RG theory [14, 99],

$$(13) \quad \sigma(H, T) - \sigma(0, T) = -0.084 \frac{e^2}{\pi\hbar} \gamma_2 (\gamma_2 + 1) \left(\frac{g\mu H}{kT} \right)^2,$$

we conclude that the interaction parameter γ_2 (roughly, $\propto |F_0^a|$) *decreases* as temperature decreases. This result contradicts the main idea of the two-parameter scaling, where γ_2 is expected to increase and diverge as the length scale increases (i.e. T decreases). It is important also that the direct SdH measurements in low H_{\parallel} fields do not confirm strong T -dependence of F_0^a [77]. It might be that the critical regime where the RG-description is applicable is much narrower, than the range of densities ($\approx 20\%n_c$) where the $\rho(T)$ data exhibit a pronounced maximum. If this is the case, the $\rho(H_{\parallel}, T)$ analysis should be restricted also to much lower magnetic fields. We note that right at the transition and for lower densities $n \leq n_c$, the $\rho(T)$ displays an approximately T -activated exponential dependence $\rho(H, T) \propto \exp(\Delta/T)$ with $\Delta \propto (n - n_c(H)) \propto H$ [63]; the resulting dependence $\exp(H/T)$ is not easy to distinguish from the $(H/T)^2$ -dependence.

In refs. [63, 59, 96, 73], it was suggested that the transport in the critical regime is driven not only by “universal” effects of interactions among the itinerant electrons, but

also by interactions of itinerant electrons with localized ones, the latter issue (short-range aspect of interactions) is missing in most of the existing theories.

Homogeneity. There is an important issue, whether or not a spatial inhomogeneity develops in the 2D electron system as the carrier density decreases; such inhomogeneity was suggested to cause the percolation-type MI-transition [100, 101]. Though this issue was considered in a number of theoretical papers, there is no clear experimental evidence for such mechanism to be the major driving force in the experimentally observed phenomena in high mobility samples.

5. – Summary

I will present below a brief pedagogical summary of the issues which has been learnt in the considered field, and those which still remain unanswered.

- For high densities and high conductivities, in the ballistic regime $T\tau \gg 1$, it is now widely accepted that the metallic T -dependence of the conductivity is a finite temperature effect caused by $e - e$ interactions. However, (i) in the diffusive regime $T\tau \ll 1$, a thorough comparison with theory is missing; some data reveal a substantial disagreement [73] with theory, which may be attributed, e.g., to the intervalley scattering; (ii) none of experimental sets of $\rho(H, T)$ -data has demonstrated an agreement with interaction theory for both zero and non-zero in-plane field.
- So far, all data find a reasonable explanation within the FL theory; no clear signature of the non-FL behavior was detected. Within the frameworks of the FL, the 2D metal will not survive upon cooling and will transform to the localized state.
- For low densities, in the critical regime $n \approx n_c$, the observed one-parameter critical behavior of conduction $\sigma(T)$ seems to be characteristic of intermediate temperatures only. For much lower temperatures, in the diffusive regime, the potential critical behavior remains unexplored.
- Both, the growth of the spin susceptibility as density decreases and the non-monotonic T -dependence of resistivity in the critical regime $n \approx n_c$, are in a qualitative agreement with the RG theory. However, some other experimental results disagree with the RG theory; the discrepancy requires more detailed analysis of the experimental data and the RG scenario of the MIT in 2D.
- The behavior of the renormalized spin susceptibility and effective mass for 2D holes at high r_s values remains puzzling; the reason of the discrepancy between the 2D hole and electron systems is not clear.
- By now, there is no complete theory which would include short-range and high-energy aspects of $e - e$ interactions, inter-valley scattering, etc.

- The interesting ideas of the potential spontaneous polarization transition in the spin or valley system require investigations at substantially higher r_s values.
- The detailed experimental verification of the possible formation of local moments, melting of the Wigner glass and other non-FL scenarios is currently lacking.

* * *

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